BACCALAURÉAT GENERAL

EPREUVE SPECIFIQUE

DES SECTIONS EUROPENNES

MATHEMATIQUES – ANGLAIS

LES CANDIDATS RESTITUENT LES TEXTES A L'ISSUE DE LEUR EPREUVE.

L'usage de la calculatrice est autorisé.
L'usage du dictionnaire est non autorisé.

Sequence

PART 1

While the story behind the problem changes from person to person, the fable usually follows the same idea:
When the creator of the game of chess showed his invention to the ruler of the country, the ruler was so pleased that he gave the inventor the right to name his prize for the invention. The man, who was very wise, asked the king this: that for the first square of the chess board, he would receive one grain of wheat (in some tellings, rice), two for the second one, four on the third one, doubling the amount each time. The ruler quickly accepted the inventor's offer, and ordered the treasurer to count and hand over the wheat to the inventor. However, when the treasurer took more than a week to calculate the amount of wheat, the ruler asked him for a reason for his tardiness. The treasurer then gave him the result of the calculation, and explained that it would be impossible to give the inventor the reward. The ruler then, to get back at the inventor who tried to outsmart him, cut off the inventor's head to discourage such trickery.

wikipedia the free encyclopedia

Questions

1.
   a. Let's consider this sequence, that has nothing to do with the previous text : 0 ; 2 ; 6 ; 12;20;30;42;56...

      Work out the differences between two following terms. What do you observe ?

   b. Find the missing numbers in this sequence : 0 ; 2 ; 6 ; 12 ; 20 ; 30 ; 42 ; 56 …
2. Can you explain why ?
3. Is this sequence arithmetic ? geometric ? Explain.
4. $u_n = n^2 - n$ with $n > 0$ **, *n*** an integer.

   Check that this relation is true for the fourth term.
5. Why all the terms of this sequence are even ?

Pythagorean Triple

Part 1 :

Pythagoras of Samos (579-475 BC), is often described as « the first pure mathematician ». Samos was a principal commerce center of Greece and is situated on the island of Samos in the Aegean Sea. The ancient town of Samos now lies in ruins. Mysteriously, none of Pythagoras's writings still exist, and we know very little about his life. He founded a mathematical society in Croton, in what is now Italy, whose members discovered irrational numbers and the five regular solids. They proved what is now called the Pythagorean Theorem, although it was discovered and used 1000 years earlier by the Chinese and Babylonians. Some historians believe that the ancient Egyptians also used a special case of the property to construct right triangles. A Pythagorean triple consists of three positive integers a, b, c such that $a^2+b^2=c^2$.
Such a triple is commonly written *(a,b,c).* Moreover such a triple *(a,b,c)* is said to be primitive if and only if *a* and *b* are coprime and one of them is even.

Questions

1. What states the Pythagorean Theorem?
2. Is (3,4,5) is a Pythagorean triple?
3. Let *k* be a positive integer, prove that if *(a, b, c)* is a Pythagorean triple, then *(ka, kb, kc)* is another one. What can you say about the number of Pythagorean triples?
4. Assume *m* and *n* are positive integers, with *m* > *n* . Let's say that $a=b=2m$ and $c=m^2+n^2$.

   Show that *(a,b,c)* is a Pythagorean triple.
5. Thanks to question 4, give an example of a primitive Pythagorean triple (different from (3,4,5)).

Weather forecasting: pushing the horizon

As natural science has revealed, our ability to predict is limited by the nature of complex systems. Weather forecasts, for example, are quite accurate a day or two out. Three or four days out, they are less accurate. Beyond a week, we might as well flip a coin.
As scientists learn more about weather, and computing power and sophistication grow, this forecasting horizon may be pushed out somewhat. But there will always be a point beyond which meteorologists cannot see, even in theory.

(Philip Tetlock and Dan Gardner www.forbes.com — march 2011 issue)

Questions

Wet or Dry ? Play the forecaster !
We want to forecast the probability that tomorrow (day 2) is either dry (D_2) or wet (W_2), given that today (day 1) is dry (D_1) or wet (W_1).

1. We know the conditional probability $P_{D_1}(W_2)=0.2$ that reads "there is a 20% chance that tomorrow will be wet, knowing that today is dry". How would you read $P_{W_1}(D_2)=0.3$?
2. Sketch the probability tree diagram that describes the situation.
3. The so-called « persistence method» states that "tomorrow usually is the same as today". What this assumption a consistent one?
4. Let the probability of having dry conditions at day 1 be $P(D_1)=0.4$. What can we predict about the weather for day 2?

A Heated Debate: Celsius versus Fahrenheit

Though most of the world uses the Celsius scale, the Fahrenheit scale may be better suited to meteorology. For one thing, it is more precise and less coarse simply because each degree represents a smaller interval. More importantly, the range in temperature from 0 to 100 degrees Fahrenheit almost perfectly demarcates the extremes found in the climates of the United States and Europe; it seldom gets any hotter or colder. However, the advantages of the Celsius scale in other aspects will win out in the end. For instance, a Celsius degree is the same "size" as a degree Kelvin, making conversions and calculations much easier. Zero on the Kelvin scale equals absolute zero—the coldest temperature theoretically possible.

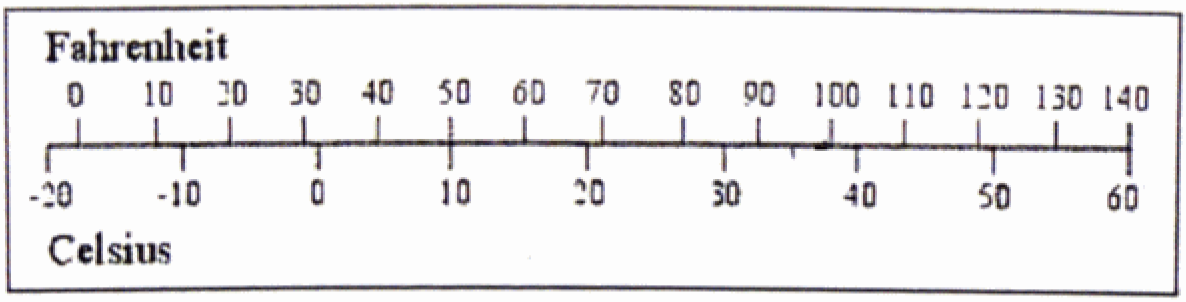
(From Tying Down the Wind, by Eric Pinder)

Questions

For any given temperature:

- let x be its numerical value in Celsius degrees,
- and y be its measure in Fahrenheit degrees.

A double scale thermometer is shown below:



1. Give an approximate value in °F of the temperature fora freezer, then for a living human body.
2. Find in this diagram at least two exact matches between values in °C and °F.
3. The conversion law from °C to °F is linear, that is it reads y = *a.x* + *b* where *a* and *b* are fixed parameters.
   a. Explain why **140 = 60a+b**.
   b. Can you find a second equation of the same kind?
   c. Solve the system formed by the two previous equations to get the conversion formula.
4. I'm measuring the local air temperature at midday in August somewhere in the world. My double scale thermometer is displaying the same numerical value both in °C and °F units. Am I in Hawaii?

Skating

Jumps are one of the most important elements of figure skating.
Jumps involve the skater leaping into the air and rotating rapidly to land after completing one or more rotations. There are many types of jumps, identified by the way the skater takes off and lands, as well as by the number of rotations that are completed. Under-rotations or using the incorrect edge will lower the jump's score. The judges also look at height, speed, and ice coverage.
Jumps can be rotated in clockwise or counterclockwise direction. Most skaters are counterclockwise jumpers. For clarity, all jumps will be described for a skater jumping counter-clockwise.
There are six jumps in figure skating that count as jump elements. All six are landed on one foot on the right back outside edge (with counterclockwise rotation, for single and multi-revolution jumps), but have different takeoffs, by which they may be distinguished.
The two categories of jumps are toe jumps ande edge jumps.

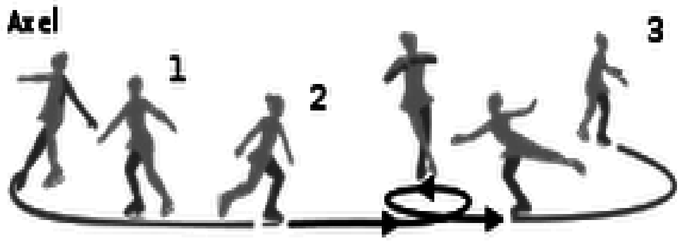


Figure skating from Wikipedia

Questions

A skater is taking part in a competition.
He's bothered about two of his jumps. He manages the first jump 95 times over 100. If he fails the first jump, he gets emotive and fails the second jump 3 times over 10. If he manages the first jump, he also manages the second one 90 times over 100.
We note R_1 the event " the skater manages the first jump " and R_2 the event " the skater manages the second jump ".

1. Summarize this situation with a tree diagram.
2.
   a. Describe the event $R_1 \cap \overline{R_2}$.
   b. Compute the probability of this event.
3. Prove that the probability of R_2 is equal to 0.89.
4. The skater wins 10 points for each jump that he succeeds. We note T the total number of points he wins for these two jumps.
   a. What are the possible values for T?
   b. Compute the probability of each of the possible values of T. Give the results in a table.
   c. What is the average number of points he can expect for these two jumps?

The Nine Chapters

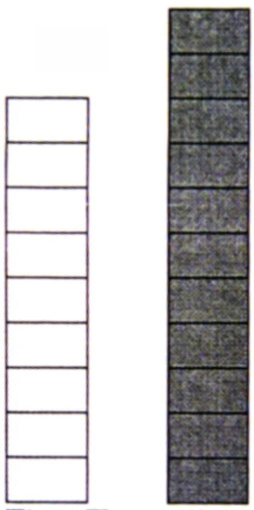
« The Nine Chapters on the Mathematical Art » is one of the earliest surviving mathematical texts from China, composed by several generations of scholars from the 10th to the 2nd century BC. It lays out an approach to mathematics that centres on finding the most general methods of solving problems, which may be contrasted with the approach common to ancient Greek mathematicians, who tended to deduce propositions from an initial set of axioms. Entries in the book usually take the form of a statement of a problem, followed by the statement of the solution, and an explanation of the procedure that led to the solution.

Sources :
http://en.wikipedia.org/wiki/The_Nine_Chapters..on.the_Matheatical_Art
http://www.gap-system.org/-history/HistTopics/Nine_chapters.html

Questions

Here is an exercise taken from this famous book :
(Problem 7-19) There are two piles, one containing 9 gold coins and the other 11 silver coins. The two piles of coins weigh the same. One coin is taken from each pile and put on top of the other. Now the pile of mainly gold coins weighs 13 units less than the pile of mainly silver coins. Find the weight of a silver coin and of a gold coin.
In order to solve this problem, let's cal ***x*** the weight of a golden coin and y the weight of silver coin.

a. Write an equation describing what happens with the first two piles.
b. Complete the following drawing to show the composition of the two new piles of coins.
c. Then write another equation describing what happens with these two piles.
d. Solve the system made of those two equations.
e. Give your conclusion



First Two piles

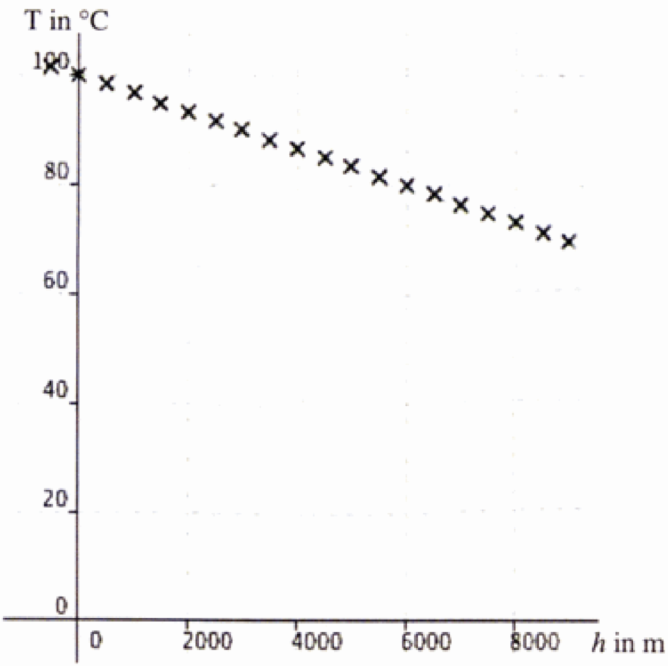
Boiling water

Boiling water is very easy to do, but it is crucial to many meals, such as cooking rice and Pasta. Water will boil at high altitudes, but it isn't as hot as boiling water at sea level. This is because the air pressure is lower at high elevations. Boiling occurs when the water is hot enough to have the same pressure as the surrounding air, so that it can form bubbles. At high altitudes, air pressure is lower than at sea level, so the water doesn't have to get so hot to get to boiling.
Because the temperature of the boiling water is lower at high elevations than at sea level, it takes longer to cook at higher altitudes than at sea level. The speed that food cooks is not related to the time it takes to boil. Adding a little salt to the water will cause the water to boil at a slightly higher temperature which can be helpful while cooking especially at high altitudes.

From the website « http://whatscookingamerica.net »

Exercise

	0	100	200	300	400
-500	101.7	101.3	101.0	100.7	100.3
0	100.0	99.7	99.3	99.0	98.7
500	98.3	98.0	97.7	97.3	97.0
1000	96.7	96.3	96.0	95.7	95.3
1500	95.0	94.7	94.3	94.0	93.7
2000	93.3	93.0	92.7	92.3	92.0
2500	91.7	91.3	91.0	90.7	90.3
3000	90.0	89.7	89.3	89.0	88.6
3500	88.3	88.0	87.6	87.3	87.0
4000	86.6	86.3	85.9	85.6	85.3
4500	84.9	84.6	84.3	83.9	83.6
5000	83.3	82.9	82.6	82.2	81.9
5500	81.6	81.2	80.9	80.6	80.2
6000	79.9	79.5	79.2	78.9	78.5
6500	78.2	77.8	77.5	77.2	76.8
7000	76.5	76.1	75.8	75.5	75.1
7500	74.8	74.4	74.1	73.8	73.4
8000	73.1	72.7	72.4	72.0	71.7
8500	71.4	71.0	70.7	70.3	70.0
9000	69.6	69.3	69.0	68.6	68.3



The temperature of the boiling water depends on the altitude :
For instance, 6700=6500+200 so the water boiling temperature is 77.5°C at 6.700 m.

1. What is the water boiling temperature at the summit of the « Mont Blanc » which is at the altitude of 4.800 m.
2. If the boiling temperature is between 94°C and 95°C, what is possible to say about the altitude ?
3. We would like to find a function f that gives the boiling temperature knowing the altitude : so T=f(h) where T is in °C and h in m.

   Do you think it is better to look for f as a linear function or as quadratic function ?
4. Find a formula for f.
5. Using your formula, calculate the boiling temperature at the altitude of 12,300 m. Actually, the boiling temperature at 12,30^m is 58.2°C. What can you say about that ?

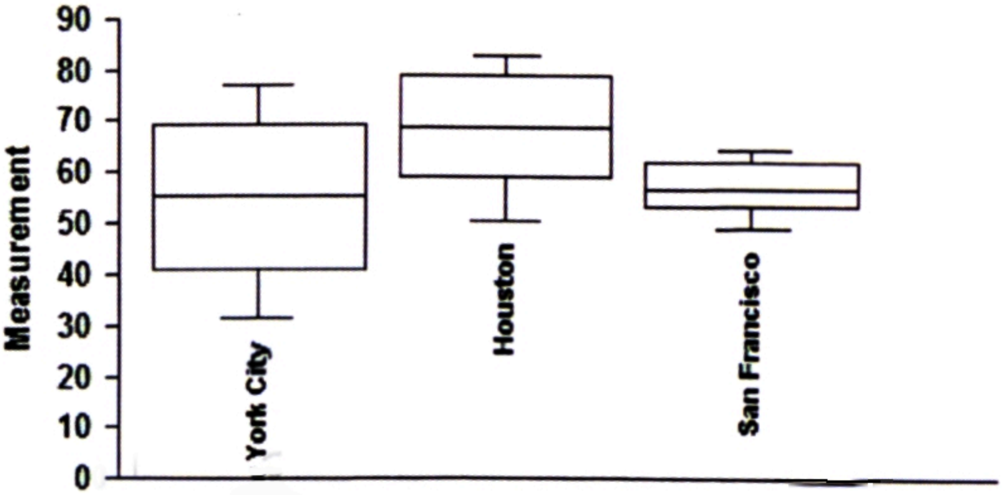
Statistics

As a chemist-turned-topologist-turned statistician, John Wilder Tukey, 1915- 2000, played a key role in the development and study of statistics in the mid 1900's.
The field of statistics has benefited tremendously from his contributions.
Two of his specific contributions are the Box-and-Whisker Plot and the Stem-and-Leaf Diagram. Box-and-Whisker Plots were invented by Tukey as a means to display groups of data and to compare them.
Tukey also invented the Stem-and-Leaf Diagram as a means to summarize the shape of a set of data (the distribution) and to provide extra detail regarding individual values.

Adapted from an article published by the School of Statistics, University of Minnesota.

Exercise

Here is a chart where each Box-and-Whisker Plot shows the average monthly temperature in three cities during the year 2010.



from 2010 BPI Consulting

1. Compare the three cities according to their average temperatures with the help of these three Boxes (center, spread, hottest, coldest,variations …)

Data was collected from 30 students (15 males and 15 females) on the number of text messages they had sent in the previous 24 hours. The set of data collected is given below :

Males

1	2	4	5	5	6	7	8	9	9	10	10	11	11	71

Females

8	9	11	11	12	15	16	18	18	18	20	21	27	34	41

2. Draw on the same diagram the Box-and-Whisker-Plots showing the number distribution for male and female students.
3. Compare these two data sets.

Eratosthenes of Cyrene

A postulate of spherical astronomy is that the Earth is a small point in relation to the heavenly bodies. From this Eratosthenes assumed, the Sun's rays striking the Earth were parallel over its entire surface. Working in Syene and Alexandria, which Eratosthenes believed were on the same meridian, he estimated the distance between the cities to be about 5,000 stades (about 500 geographical miles or 800km). At summer solstice, at noon, the Sun cast no shadow in Syene, but in Alexandria a shadow was visible. Using a vertical stick, Eratosthenes measured the shadow's angle (a in fig1.) to be about one-fiftieth of a circle.

AS = 5,000 stades.

The lines OB and CS are parallel lines because they follow sun rays. [OA] is the shadow of the stick [AB].

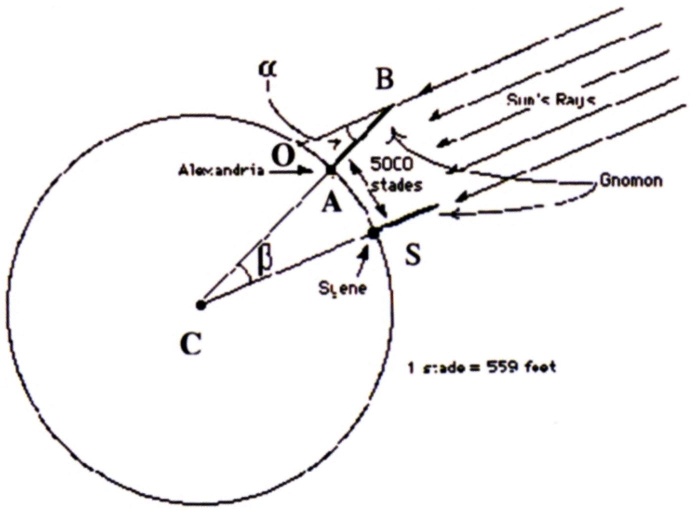


Fig 1.

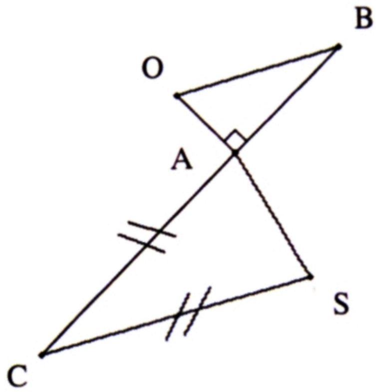


Fig.2

http://share2.esd 105.wednet.edu/jmcald/Aristarchus/eratosthenes.html

Exercise

1. What is the measure of angle $\widehat{OBA}$?
2. Prove that $\widehat{OBA} = \widehat{ACS}$.
3. Calculate the perimeter and then the radius of the Earth.
4. Compare this result with 6.371 km, the actual value of the radius of the Earth